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Radial excitations of heavy-light mesons

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Abstract. The recent discovery of D_s states suggests the existence of radial excitations. Our semirelativistic quark potential model succeeds in reproducing these states within one to two percent of accuracy compared with the experiments, $D_{s0}(2860)$ and $D_s^*(2715)$, which are identified as 0^+ and 1^- radial excitations ($n=2$). We also present calculations of radial excitations for B/B_s heavy mesons. The relation between our formulation and the modified Goldberger-Treiman relation is also described.

PACS. 12.39.Hg Heavy quark effective theory – 12.39.Pn Potential models – 12.40.Yx Hadron mass models and calculations – 14.40.Lb Charmed mesons

1 Introduction

BaBar has recently announced the discovery of a new D_s state, which seems to be the $c\bar{s}$ state [1], $D_{s0}(2860)$. Subsequent to this experiments Belle has observed a new state of $D_s^*(2715)$ whose spin and parity are determined to be 1^- [2].

We were the first in predicting 0^+ and 1^+ states of D_s and D particles, $D_{s0}(2317)$, $D_{s1}(2460)$, $D_0^*(2308)$, and $D_1'(2427)$, [3], and we have also succeeded in reproducing higher resonances of B/B_s particles, $B_1(5720)$, $B_2^*(5745)$, and $B_{s2}^*(5839)$ [4], using our semirelativistic quark potential model. Our model succeeds in lowering 0^+ and 1^+ states of D_s so that these mass values are below DK/D^*K threshold while other models do not. This model respects both heavy-quark symmetry and chiral symmetry in a certain limit of parameters [5], which relates our formulation with the idea of the modified Goldberger-Treiman relation proposed in refs. [6,7]. Hence it is natural to try to explain the newly discovered D_s states by using our semirelativistic model and we are again successful in reproducing these states discovered by BaBar and Belle.

To interpret the state $D_{s0}(2860)$, there are arguments that this $c\bar{s}$ state is explained to be a scalar by a coupled channel model [8], or that it is a $J^P=3^-$ state [9], or that it can be explained by using a phenomenological interaction term like our quark potential model [10], or that it can be analyzed by using the $^3P_0$ model [11].

Starting from the astonishing discovery of D_{sJ} particles with narrow decay width by BaBar and CLEO, and confirmed by Belle, a series of successive experiments on the spectrum of a heavy-light system, *i.e.*, heavy mesons, heavy quarkonium, and heavy baryons, stimulates theorists to explain all these spectra as well as their decay modes. See the recent reviews of refs. [12,13]. It seems that a new era of spectroscopy is opening, which is challenging to theorists to solve these spectra at the same time.

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Table 1. Most optimal values of the parameters.

Parameters	$\alpha_s^{n=2}$	a (GeV^{-1})	b (GeV)
	0.344	1.939	0.0749

$m_{u,d}$ (GeV)	m_s (GeV)	m_c (GeV)	m_b (GeV)
0.0112	0.0929	1.032	4.639

2 Numerical calculation

Our model starts from a Hamiltonian with a scalar confining potential together with a Coulombic vector potential, we expand the whole system, *i.e.*, the Hamiltonian, the wave function, and the eigenvalue is expanded in $1/m_Q$, and we solve the equations order by order consistently. In the actual numerical calculation, we expand the wave function in a power series of relative coordinate with some weighting exponential times power functions. The wave function has positive components of a heavy quark due to the lowest-order constraint when expanding in $1/m_Q$

Table 2. $D_s(n=2)$ meson mass spectra (first order). Units are in MeV.

$^{2s+1}L_J(J^P)$	M_0	c_1/M_0	p_1/M_0	n_1/M_0	M_{calc}	M_{obs}
$^1S_0(0^-)$	2328	1.006×10^{-1}	0.919×10^{-1}	8.695×10^{-3}	2563	–
$^3S_1(1^-)$		1.830×10^{-1}	1.824×10^{-1}	5.744×10^{-4}	2755	2715
$^3P_0(0^+)$	2456	1.553×10^{-1}	1.470×10^{-1}	8.245×10^{-3}	2837	2856
"$^3P_1$"(1^+)		2.551×10^{-1}	2.543×10^{-1}	7.667×10^{-4}	3082	–
"$^1P_1$"(1^+)	2585	1.969×10^{-1}	1.966×10^{-1}	2.531×10^{-4}	3094	–
$^3P_2(2^+)$		2.209×10^{-1}	2.209×10^{-1}	5.070×10^{-7}	3157	–
$^3D_1(1^-)$	2391	8.605×10^{-1}	8.600×10^{-1}	5.016×10^{-4}	4449	–
"$^3D_2$"(2^-)		-4.287×10^{-1}	-4.287×10^{-1}	5.482×10^{-7}	1366	–

Table 3. $D(n=2)$ meson mass spectra (first order). Units are in MeV.

$^{2s+1}L_J(J^P)$	M_0	c_1/M_0	p_1/M_0	n_1/M_0	M_{calc}	M_{obs}
$^1S_0(0^-)$	2241	1.078×10^{-1}	0.975×10^{-1}	1.038×10^{-2}	2483	–
$^3S_1(1^-)$		1.917×10^{-1}	1.910×10^{-1}	6.882×10^{-4}	2671	–
$^3P_0(0^+)$	2418	1.540×10^{-1}	1.444×10^{-1}	9.621×10^{-3}	2791	–
"$^3P_1$"(1^+)		2.493×10^{-1}	2.488×10^{-1}	5.352×10^{-4}	3021	–
"$^1P_1$"(1^+)	2491	2.076×10^{-1}	2.070×10^{-1}	5.956×10^{-4}	3008	–
$^3P_2(2^+)$		2.319×10^{-1}	2.318×10^{-1}	1.101×10^{-4}	3069	–
$^3D_1(1^-)$	2280	1.869×10^{-1}	1.863×10^{-1}	5.418×10^{-4}	2706	–
"$^3D_2$"(2^-)		2.100×10^{-1}	2.099×10^{-1}	1.203×10^{-4}	2759	–

Table 4. $B(n=2)$ meson mass spectra (first order). Units are in MeV.

$^{2s+1}L_J(J^P)$	M_0	c_1/M_0	p_1/M_0	n_1/M_0	M_{calc}	M_{obs}
$^1S_0(0^-)$	5849	0.919×10^{-2}	0.831×10^{-2}	8.849×10^{-4}	5902	–
$^3S_1(1^-)$		1.634×10^{-2}	1.629×10^{-2}	5.867×10^{-5}	5944	–
$^3P_0(0^+)$	6025	1.375×10^{-2}	1.289×10^{-2}	8.590×10^{-4}	6108	–
"$^3P_1$"(1^+)		2.226×10^{-2}	2.221×10^{-2}	4.779×10^{-5}	6160	–
"$^1P_1$"(1^+)	6098	1.886×10^{-2}	1.881×10^{-2}	5.412×10^{-5}	6213	–
$^3P_2(2^+)$		2.108×10^{-2}	2.107×10^{-2}	1.001×10^{-5}	6227	–
$^3D_1(1^-)$	5888	1.610×10^{-2}	1.605×10^{-2}	4.668×10^{-5}	5982	–
"$^3D_2$"(2^-)		1.809×10^{-2}	1.808×10^{-2}	1.037×10^{-5}	5994	–

Table 5. $B_s(n=2)$ meson mass spectra (first order). Units are in MeV.

$^{2s+1}L_J(J^P)$	M_0	c_1/M_0	p_1/M_0	n_1/M_0	M_{calc}	M_{obs}
$^1S_0(0^-)$	5936	0.878×10^{-2}	0.802×10^{-2}	7.588×10^{-4}	5988	–
$^3S_1(1^-)$		1.597×10^{-2}	1.592×10^{-2}	5.013×10^{-5}	6031	–
$^3P_0(0^+)$	6063	1.399×10^{-2}	1.325×10^{-2}	7.429×10^{-4}	6148	–
"$^3P_1$"(1^+)		2.299×10^{-2}	2.292×10^{-2}	6.908×10^{-5}	6202	–
"$^1P_1$"(1^+)	6193	1.828×10^{-2}	1.826×10^{-2}	2.351×10^{-5}	6306	–
$^3P_2(2^+)$		2.052×10^{-2}	2.052×10^{-2}	4.709×10^{-8}	6320	–
$^3D_1(1^-)$	5999	7.631×10^{-2}	7.627×10^{-2}	4.449×10^{-5}	6456	–
"$^3D_2$"(2^-)		-3.802×10^{-2}	-3.802×10^{-2}	4.861×10^{-8}	5770	–

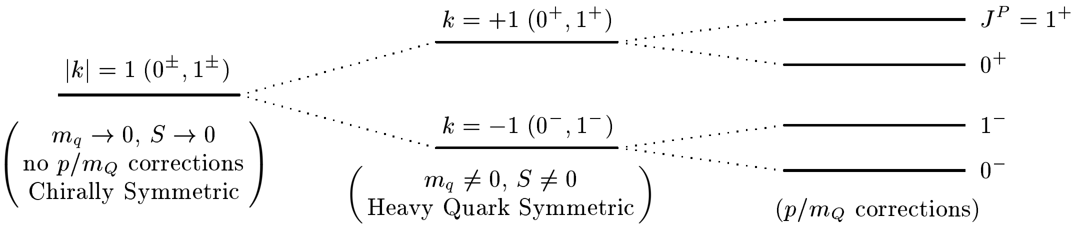

Fig. 1. Procedure how the degeneracy is resolved in our model. A quantum number k is defined in the main text.

and four components of a light antiquark. Thus, the lowest eigenfunction is a two-by-four matrix. Other than the angular part, its radial part can be written as:

$$u_k(r),\ v_k(r) \sim w_k(r) \left(\frac{r}{a}\right)^\lambda \exp\left[-(m_q+b)r - \frac{1}{2}\left(\frac{r}{a}\right)^2\right],$$

where k is the quantum number of the operator, $-\beta_q(\boldsymbol{\Sigma}_q \cdot \boldsymbol{L}+1)$ with $\boldsymbol{\Sigma}_q$ light-quark spin and $\boldsymbol{L}$ light-quark angular momentum, which distinguishes uniquely each state [3], for instance, $k=-1$ for a spin multiplet ($J^P = 0^-, 1^-$), $k=+1$ for $(0^+, 1^+)$, etc., $u_k(r)$ and $v_k(r)$ are the upper and lower components of the radial wave functions, and $\lambda = \sqrt{k^2 - (4\alpha_s/3)^2}$, where a and b are included in a scalar potential as $S(r) = r/a^2 + b$, α_s is the strong coupling, and $w_k(r)$ is the finite series of a polynomial in r, $w_k(r) = \sum_{i=0}^{N-1} a_i^k (r/a)^i$, which takes different coefficients for $u_k(r)$ and $v_k(r)$. In the actual calculation, we have used $N=7$. Hence we can in principle obtain seven different radial excitations.

In this paper, we calculate the most optimal values of the parameters so that the recently discovered and known $n=2$ (the first radial excitation) D_s particles are all fitted well around one percent of accuracy compared with the experiments. Here only the strong coupling α_s is modified and other parameters are kept the same as in [4], in which we have obtained $\alpha_s = 0.261$ both for D and D_s. These are presented in table 1 at the first order of calculation in p/m_Q with p being the internal quark momentum and m_Q the heavy-quark mass.

With these values of the parameters, we obtain $n=2$ masses of D_s and D at the same time. The results are shown in table 2 for D_s. We also predict $n=2$ states for D, B, and B_s states, which are shown in tables 3, 4, and 5 assuming the same strong coupling α_s for D_s, which may actually be different for B/B_s particles. In these tables, p_i and n_i are the i-th order positive and negative component contributions of a heavy quark, respectively, and $c_i = p_i + n_i$. When one carefully looks at these tables, one notices that the values of the higher states $^3D_1$ and "$^3D_2$" are not reliable even though we have listed them in the tables for consistency with the former calculations.

3 Mass gap

Noticing that the mass gaps between spin multiplets, $(0^-, 1^-)$ and $(0^+, 1^+)$ for D_{sJ} mesons, are almost equal to each other, the modified Goldberger-Treiman relation is proposed by Bardeen *et al.* [6,7] to understand the facts, *i.e.*, the underlying physics might be chiral physics. In other words the mass gap is due to the chiral symmetry breakdown which is expressed by this relation. Further they have assumed that the hyperfine splittings due to breakdown of heavy-quark symmetry (inclusion of $1/m_Q$ corrections) are the same within two spin multiplets, $(0^-, 1^-)$ and $(0^+, 1^+)$, so that the mass gap is not affected by this hyperfine splitting.

Table 6. Theoretical mass gap. Values in brackets are experiments. Units are in MeV.

Mass gap ($n=1$)	D	D_s	B	B_s
0^+–0^-	414 (441)	358 (348)	322	239
1^+–1^-	410 (419)	357 (348)	320	242

($n=2$)	D	D_s	B	B_s
0^+–0^-	308	274	206	160
1^+–1^-	350	327	216	171

In our formulation this is explained in fig. 1 [5]. Heavy-quark symmetry reduces the original Hamiltonian into the one without spin structure after projecting wave functions into the positive and negative components of a heavy quark [3,5]. The chiral symmetry of a light quark is realized by taking a limit of $m_q \to 0$ and $S(r) \to 0$, in which case all the members of two spin multiplets, $(0^-, 1^-)$ and $(0^+, 1^+)$, are degenerate. When the light-quark mass and a scalar potential are turned on, then the degeneracy due to chiral symmetry is resolved and the mass gap corresponding to the modified Goldberger-Treiman relation is given by

$$\Delta M = M_0(k=+1) - M_0(k=-1),$$

where $M_0(k)$ is a degenerate mass for a quantum number k, which appears in tables 2–5 to distinguish states. This is described as $\Delta M = \tilde{g}_\pi f_\pi / G_A$ in [6]. They have assumed dominant interaction terms for hyperfine splitting due to $1/m_Q$ corrections so that mass gaps between 0^+–0^- and 1^+–1^- are almost equal to each other, which seems to hold in our formulation. Our hyperfine splittings due to $1/m_Q$ to this mass gap ΔM are calculated and we have added those to degenerate mass gaps between 0^+–0^- and 1^+–1^- for D, D_s, B, and B_s with $n=1$, which are given in table 6. Our dynamical calculation supports the assumption that the mass gaps between 0^+–0^- and 1^+–1^- are almost equal to each other in the case of $n = 1$. In the second row of the same table values for $n = 2$ are given, which is not as good as the case for $n = 1$.

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